On Tachyon Resolution

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In this paper, we attempt to clarify some purported misunderstandings concerning tachyons. We also offer a possibility that tachyons may not actually be superluminal although they appear to be so to us.

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I. INTRODUCTION

There appear to be some misunderstandings concerning superluminal motions or tachyons (here we mean them by generic hypothetical superluminal particles) in physics literature including textbooks. For example, a claim that superluminal motions violate special relativity, that tachyons have an imaginary rest mass, that tachyons would satisfy the usual energy-momentum relation, or that Tachyons would violate causality. These purported misunderstandings are mainly due to applying incorrect Lorentz transformation (the usual one we see in textbooks that is actually meant for subluminal motions). In the next section, we introduce the Lorentz transformation for superluminal motions and use it to clarify such misunderstandings. In the final section, we offer a possibility that tachyons may not actually be superluminal although they appear to be so to us. This requires a hypothesis that there is another universe which has a different signature than that of our universe.

II. THE LORENTZ TRANSFORMATION FOR SUPERLUMINAL MOTIONS

Let us denote by $\mathbb{R}^{3+1}$ the Minkowski space with the Lorentz-Minkowski metric

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 \tag{1}$$

Let us denote (ct', x') be a frame that travels at a constant speed. Then by a Lorentz boost we have

$$\begin{aligned} ct' &= ct\cosh\phi - x\sinh\phi \\ x' &= -ct\sinh\phi + x\cosh\phi \end{aligned} \tag{2}$$

For a superluminal motion, we consider a point defined by $ct' = 0$. Then from (2) we find that its speed v is given by

$$v = \frac{x}{t} = c\frac{\cosh\phi}{\sinh\phi} = c\coth\phi \tag{3}$$

where $\coth\phi > 1$. $\sinh\phi$ can be written as

$$\sinh\phi = \frac{1}{\sqrt{\frac{v^2}{c^2} - 1}}$$

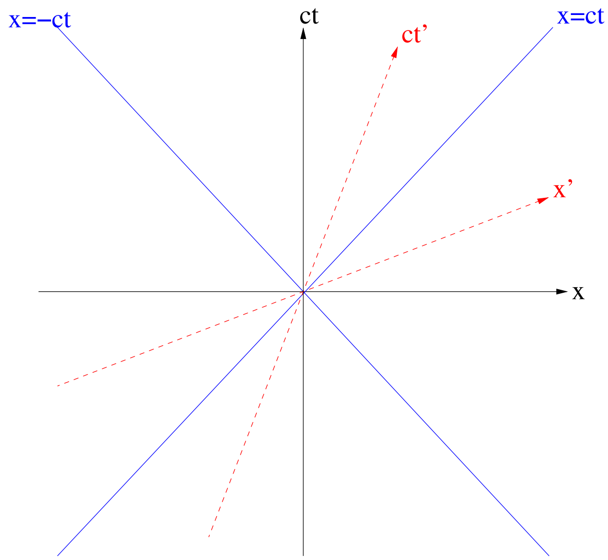

FIG. 1. Lorentz boost

and we obtain the Lorentz transformation for a superluminal motion

$$t' = \frac{t - \frac{x}{v}}{\sqrt{1 - \frac{c^2}{v^2}}}$$
$$x' = \frac{x - \frac{c^2}{v}t}{\sqrt{1 - \frac{c^2}{v^2}}} \tag{4}$$

The transformation in (4) is indeed a Lorentz transformation as it is an isometry, i.e. the Lorentz-Minkowski metric (1) is invariant under (4).

Now we study the relativistic effects on length, time and mass for a superluminal motion.

A. Lorentz Contraction for a Superluminal Motion

Let an observer in frame S at rest measure the length of a rod as $l = x_2 - x_1$ at time t. If, for the frame S' moving at the constant speed $v > c$, the coordinates of the front and the back ends of the rod are x'_1 and x'_2, respectively at time t, then by Lorentz transformation (4), we have

$$x'_1 = \frac{x_1 - \frac{c^2}{v}t}{\sqrt{1 - \frac{c^2}{v^2}}}$$
$$x'_2 = \frac{x_2 - \frac{c^2}{v}t}{\sqrt{1 - \frac{c^2}{v^2}}}$$

Let $l_0 = x'_2 - x'_1$. Then

$$l = l_0\sqrt{1 - \frac{c^2}{v^2}} \tag{5}$$

B. Time Delay for a Superluminal Moiton

This time, let us consider two events (t_1, x) and (t_2, x) in frame S at rest. For an observer at rest, in the frame S' that is moving at a constant speed $v > c$, the two events happened, respectively at

$$t'_1 = \frac{t_1 - \frac{x}{v}}{\sqrt{1 - \frac{c^2}{v^2}}}$$
$$t'_2 = \frac{t_2 - \frac{x}{v}}{\sqrt{1 - \frac{c^2}{v^2}}}$$

Hence, we obtain

$$t'_2 - t'_1 = \frac{t_2 - t_1}{\sqrt{1 - \frac{c^2}{v^2}}} \tag{6}$$

C. Momentum-Energy Relation for a Superluminal Motion

Suppose that the world vector $\vec{r} = (t, x, y, z)$ is spacelike. Then the infinitesimal proper time $d\tau$ of the system moving faster-than-light is given by

$$d\tau := \sqrt{\frac{v^2}{c^2} - 1}dt \tag{7}$$

where $\mathbf{v} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$. Let $\vec{v}$ be the four-velocity four-vector

$$\vec{v} = \frac{d\vec{r}}{d\tau} = \left(\frac{dt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\right) = \frac{1}{\sqrt{\frac{v^2}{c^2} - 1}}(c, \mathbf{v})$$

Then $\vec{v} \cdot \vec{v} = c^2$. The four-momentum $\vec{p}$ is given by

$$\vec{p} = m_0\vec{v} = \frac{m_0}{\sqrt{\frac{v^2}{c^2} - 1}}(c, \mathbf{v}) \tag{8}$$

and $\vec{p} \cdot \vec{p} = m_0^2c^2$. On the other hand, $\vec{p}$ can be also written as

$$\vec{p} = (mc, m\mathbf{v}) = \left(\frac{E}{c}, \mathbf{p}\right) \tag{9}$$

where $m = \frac{m_0}{\sqrt{\frac{v^2}{c^2}-1}}$ and $\mathbf{p} = (p_x, p_y, p_z)$. Comparing the first coordinates of (8) and (9), we obtain

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{\frac{v^2}{c^2} - 1}} \tag{10}$$

Now, from (8) or (9), we obtain the energy-momentum relation

$$E^2 = p^2c^2 - m_0^2c^4 \tag{11}$$

Although the rest mass of a tachyon is real, we see from (11) that tachyons cannot be at rest because $\mathbf{p} = \mathbf{O}$ leads to energy being purely imaginary. In fact, from (10), we see that the speed of a tachyon cannot be slower than the speed of light.

D. Do Tachyons Really Violate Causality?

The claim that tachyons violate causality is usually based on Richard C. Tolman's thought experiment[1] or some variations of it. In order to maintain this paper self-contained, we include Tolman's thought experiment here. Suppose a signal is being sent from a point x_A to another point x_B with a constant speed a. In an inertial frame S where x_A and x_B are at rest, the time of arrival at x_B is given by

$$\Delta t = t_B - t_A = \frac{x_B - x_A}{a}$$

In another inertial frame S' moving at a constant speed v, relative to S, the time of arrival at B is given by

$$\begin{aligned}\Delta t' &= t'_B - t'_A \\ &= \frac{t_B - \frac{v}{c^2}x_B}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_A - \frac{v}{c^2}x_A}{\sqrt{1-\frac{v^2}{c^2}}} \\ &= \frac{1 - a\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\Delta t\end{aligned}$$

Now suppose that $a > c$. Then for a certain value of v, to be specific, if v satisfies $\frac{c^2}{a} < v < c$, then $\Delta t' < 0$, i.e. causality is violated. There does not appear to be any logical fallacy in Tolman's argument. However, the physical validity of the argument hangs onto the assumption that one can use tachyons to send a signal. Tolman's thought experiment can be stated as the conditional statement "if one can use tachyons to send a signal, causality can be violated" and it is logically equivalent to its contrapositive "if causality cannot be violated, one cannot use tachyons to send a signal". Hence, Tolman's thought experiment is not actually an argument against the existence of tachyons nor is it an argument implying that one can send a signal to the past. Its logical validity only affirms that one cannot use tachyons to send a signal. However, we do not know yet any physical mechanism that prevents us from doing so.

Interestingly, causality violation can also happen when the frame S' is moving at a constant speed $v > c$. The calculation is carried out exactly the same way except that we will have to use the Lorentz transformation for superluminal motions (4). The time of arrival at B in the moving frame S' is then given by

$$\Delta t' = \frac{1 - \frac{a}{v}}{\sqrt{1 - \frac{c^2}{v^2}}} \Delta t$$

As one can immediately see, if the frame S' is moving slower than the speed a of the signal, causality is violated.

E. Mythbusting Tachyons: Summary

1. Myth: *Superluminal motions violate special relativity.* No, they do not violate special relativity. It appears that Einstein himself doubted the possibility of the FTL (Faster-Than-Light) because the relativistic kinetic energy tends to infinity as velocity approaches the speed of light[2]. Such misunderstanding is due to applying the incorrect Lorentz transformation that is meant for subluminal motions. As we have seen, there exists Lorentz transformation for superluminal motions (4) and superluminal motions are consistent with special relativity. What is not allowed, however, is a continuous transition from a subluminal motion to a superluminal motion and vice versa.

2. Myth: *Tachyons have an imaginary rest mass.* This is also a misunderstanding due to applying the incorrect Lorentz transformation that is meant for subluminal motions[3]. This is how the claim came about. From the usual relativistic energy formula

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

if $v > c$, the denominator becomes purely imaginary. In order to keep the energy E real, the rest mass m_0 is required to be purely imaginary also. The above relativistic energy formula is based on the Lorentz transformation for subluminal motions. Hence, applying superluminal case to the formula is inappropriate. We derived the correct relativistic energy formula for tachyons in (10) and there is no need to introduce an awkward and seemingly unphysical notion of imaginary rest mass.

3. Myth: *Tachyons satisfy the usual energy-momentum relation.* Not true and we derived the correct energy-momentum relation for tachyons in (11).

4. Myth: *Tachyons violate causality.* Tolman's thought experiment or some variations of it often quoted as an argument against the existence of superluminal motions or as an argument for a possibility of sending a signal to the past. Either case involves the violation of causality. However, this is not the case as we argued in the preceding subsection. Tolman's thought experiment only affirms that one cannot use Tachyons or any superluminal particles to send a signal.

Thus far, we have argued that the existence of FTL phenomena does not contradict with special relativity and there is not really any physically peculiar aspect of tachyons contrary to some beliefs. There has not been any confirmed FTL phenomenon yet. There were some reported FTL phenomena in the past that involve the speed of quasars and that of neutrinos. It turned out the alleged FTL speed of quasars was due to an optical illusion and the FTL neutrino anomaly was caused by a faulty equipment set-up. But, even if we observe a legitimate FTL phenomenon, it may not actually be FTL contrary to our observation. In the next section, we explain how such a peculiar possibility may occur.

III. TWIN UNIVERSE HYPOTHESIS AND THE RESOLUTION OF TACHYONS

For the sake of simplicity, we assume that $c = 1$ in this section. Let us start with our universe $\mathbb{R}^{3+1}$ with the Minkowski metric

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{12}$$

The Minkowski plane $\mathbb{R}^{1+1}$ with metric $-dt^2 + dx^2$ is identified with split-complex plane $\{t + xj : t, x \in \mathbb{R},\ j^2 = 1\}$. In fact, the identification is an isometry: if $\zeta = t + xj$, then $-t^2 + x^2 = -\zeta\bar{\zeta}$. The map $\zeta = t + xj \mapsto j\zeta = x + tj$ gives rise to the reflection of ζ about the

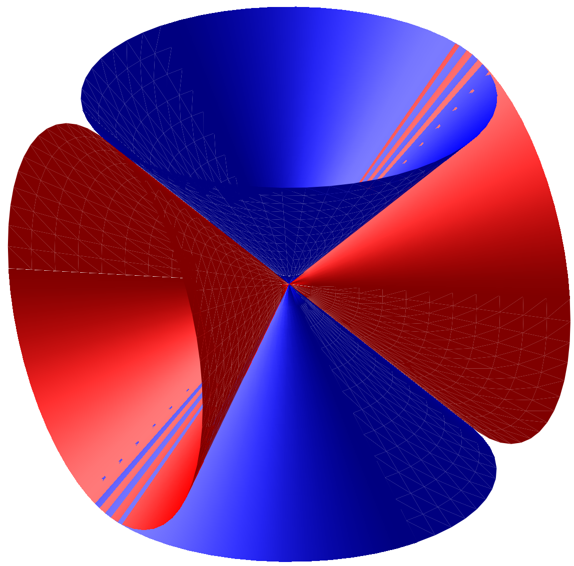

FIG. 2. The Light Cone of Our Universe (in Blue) and That of Its Counterpart Universe (in Red) with z-Coordinate Suppressed.

light ray $t = x$. Applying this reflection on the Minkowski metric (12) results in a different universe with new metric

$$
\begin{aligned}
ds^2 &= -jd\zeta\overline{jd\zeta} + dy^2 + dz^2 \\
&= d\zeta d\bar{\zeta} + dy^2 + +dz^2 \\
&= dt^2 - dx^2 + dy^2 + dz^2
\end{aligned}
$$

Figure 2 shows the light cone of our universe (in blue) and that of its counterpart (in red). As seen in Figure 2, spacelike vectors in our universe fall into the timelike region of the other universe. This means that what we observe as a superluminal motion in our universe is a subluminal motion in the other universe. To see this more clearly, let us consider a Lorentz boost in the Minkowski plane of the other universe. This transformed coordinated (x', t') under the Lorentz boost are given in terms of the original coordinates (x, t) by

$$
\begin{aligned}
x' &= x\cosh\phi - t\sinh\phi \\
t' &= -x\sinh\phi + t\cosh\phi
\end{aligned}
$$

From the second equation, a point on the x'-axis is moving at a constant speed

$$v = \frac{t}{x} = \frac{\sinh\phi}{\cosh\phi} = \tanh\phi$$

In their universe, it is reasonable to assume that the principle of special relativity holds as well. This means that $v = \tanh\phi < 1$. This motion in the other universe would, however, be observed as a superluminal motion from our universe because in our universe the speed of the point on the x'-axis is measured as

$$v = \frac{x}{t} = \frac{\cosh\phi}{\sinh\phi} = \coth\phi > 1$$

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[3] W. Greiner, *Classical Mechanics, Point Particles and Relativity* (Springer, 2004).