# Number Theory
# Problem Set 2
# Linear Combination

1. Find $x$ and $y$ such that $26x + 14y = (26, 14)$ with $x$ positive but as small as possible.

   **Answer.** $x = 6$, $y = -11$.

2. Find all solutions to $2x + 3y = 50$ in positive integers.

   **Answer.** $x = -50 + 3t$, $y = 150 - 2t$, $17 \leq t \leq 74$. You can list all these solutions in positive integers by running a simple Python program. You can download one at `https://sunglee.us/NT/diophantine.py`.

3. Prove that if $(a, b) = 1$ and $c \neq 0$, then $(ac, bc)|c$.

   **Hint.** Since $(a, b) = 1$, there exist $x, y \in \mathbb{Z}$ such that $ax + by = 1$.

4. Prove theorem 1 in the online notes on Linear Combination at `https://sunglee.us/mathphysarchive/?p=4864`.

   **Hint.** Only If part is straightforward by a well-known division property. For If part, consider Bézout's lemma.

5. Use the Euclidean algorithm method to find one solution to each equation.

   (a) $7x + 20y = 3$
   **Answer.** $x = 9$, $y = -3$.

   (b) $66x + 51y = 300$
   **Answer.** $x = 700$, $y = -900$.

6. Find all solutions to each equation with $x$ and $y$ positive.

(a) $6x + 8y = 120$

**Answer.** $x = -60 + 4t$, $y = 60 - 3t$, $16 \leq t \leq 19$.

(b) $169x + 663y = 2340$

**Answer.** $x = 6$, $y = 2$.

7. A girl spent \$100.64 on posters. Some cost \$4.98 and some \$5.98. How many did she buy?

   **Answer.** The girl bought 7 \$4.98/unit posters and 11 \$5.98/unit posters.