GROUP THEORY
PROBLEM SET 8
QUOTIENT GROUPS

(1) It was discussed in my lecture note that if $N \lhd G$, then right coset multiplication is well-defined by the equation

$$NaNb = Nab. \tag{1}$$

Show that if $N \leq G$ and right coset multiplication is well-defined by the equation (1), then $N \lhd G$.

(2) Prove that subgroup of a cyclic group is also cyclic.
Hint: This is similar to # 3 of Problem Set 3.

(3) Show that A_n is a normal subgroup of S_n and compute S_n/A_n, that is, find a known group to which S_n/A_n is isomorphic.

(4) Let $N \lhd G$. If $m = |G : N|$ then show that $a^m \in N$ for every $a \in G$.

(5) If G is a cyclic group and N is a subgroup of G, show that G/N is a cyclic group.

(6) If G is an abelian group and N is a subgroup of G, show that G/N is an abelian group.

(7) Show that if $G/Z(G)$ is cyclic, then G is abelian.

(8) If G is a group and $N \lhd G$ is such that G/N is abelian, prove that $aba^{-1}b^{-1} \in N$ for all $a, b \in G$.

(9) If G is a group and $N \lhd G$ is such that $aba^{-1}b^{-1} \in N$ for all $a, b \in G$, prove that G/N is abelian.