GROUP THEORY
PROBLEM SET 6
HOMOMORPHISMS

(1) Recall that $G \cong G'$ means that G is isomorphic to G'. Prove that for all groups G_1, G_2, G_3:
   (a) $G_1 \cong G_1$.
   (b) $G_1 \cong G_2$ implies that $G_2 \cong G_1$.
   (c) $G_1 \cong G_2$, $G_2 \cong G_3$ implies that $G_1 \cong G_3$.

(2) Let G be any group and $A(G)$ the set of all 1-1 mappings of G, as a set, onto itself. Let $a \in G$ be fixed. Define $L_a : G \longrightarrow G$ by $L_a(x) = xa^{-1}$ for each $x \in G$. Prove that:
   (a) $L_a \in A(G)$.
   (b) $L_a L_b = L_{ab}$.
   (c) The mapping $\psi : G \longrightarrow A(G)$ defined by $\psi(a) = L_a$ is a monomorphism of G into $A(G)$.

(3) Show that the inner automorphism $\varphi : G \longrightarrow G$ of a group G induced by $a \in G$ defined by

$$\varphi(x) = a^{-1}xa, \ \forall x \in G$$

is actually an isomorphism.

(4) Find an isomorphism of $(\mathbb{R}, +)$ onto $(\mathbb{R}^+, \cdot)$.

(5) If G is a finite abelian group of order n and $\varphi : G \longrightarrow G$ is defined by $\varphi(a) = a^m$ for all $a \in G$, find the necessary and sufficient condition that φ be an isomorphism of G onto itself.

(6) If G is abelian and $\varphi : G \longrightarrow G'$ is a homomorphism of G onto G', prove that G' is abelian.