GROUP THEORY
PROBLEM SET 1
BASIC NUMBER THEORY

(1) Prove the following properties. m, n, q, μ, ν in the statements (a)-(f) are all integers.
   (a) $1|n \ \forall n$.
   (b) If $m \neq 0$ then $m|0$. Explain why $0 \not| 0$.
   (c) If $m|n$ and $n|q$, then $m|q$.
   (d) If $m|n$ and $m|q$ then $m|(\mu n + \nu q) \ \forall \mu, \nu$.
   (e) If $m|1$ then $m = \pm 1$.
   (f) If $m|n$ and $n|m$ then $m = \pm n$.

(2) Prove that the integers a and b are relatively prime if and only if $1 = ma + nb$ for some m and n.

(3) Prove that if $(a, b) = 1$ and $a|bc$ then $a|c$.

(4) Prove that if b and c are both relatively prime to a, then bc is also relatively prime to a.