FUNCTIONAL ANALYSIS
PROBLEM SET 2

(1) Let (X,d) be a metric space. Let $x \in X$ and let $\epsilon > 0$ be given. Show that $B(x,\epsilon)$ is open.
(2) If x_0 is an accumulation point of a set $A \subset (X,d)$, show that any neighbourhood of x_0 contains infinitely many points of A.
(3) Let (X,d) be a metric space and $A \subset X$. Show that $\bar{A}$ is the smallest closed set containing A.
(4) Let (X,d) be a metric space and $A \subset X$. Show that $x \in \bar{A}$ if and only if $\forall$ open set $U(x)$ in X, $U(x) \cap A \neq \emptyset$.
(5) Show that $\overline{A \cup B} = \bar{A} \cup \bar{B}$ and $\overline{A \cap B} \subset \bar{A} \cap \bar{B}$. Given an example that shows $\overline{A \cap B} \neq \bar{A} \cap \bar{B}$.
(6) Let $x = (\xi_j) \in \ell^p$ with $1 \leq p < \infty$. Show that given $\epsilon > 0$ there exists a positive integer $N > 0$ such that $\displaystyle\sum_{j=N+1}^{\infty} |\xi_j|^p < \epsilon$.
(7) Show that a mapping $T : X \longrightarrow Y$ is continuous if and only if the inverse image of any closed set $F \subset Y$ is closed in X.
(8) Show that the image of an open set under a continous mapping need not be open.