FUNCTIONAL ANALYSIS
PROBLEM SET 1

(1) Let X be the set of all bounded sequences of complex numbers

$$X = \{(\xi_j) : \xi_j \in \mathbb{C},\ j = 1, 2, \cdots\}.$$

For $x = (\xi_j), y = (\eta_j) \in X$, define

$$d(x, y) = \sup_{j \in \mathbb{N}} |\xi_j - \eta_j|.$$

Show d is a metric on X.

(2) Let X be the set of continuous real-valued functions define on the closed interval $[a, b]$. Let $x, y : [a, b] \longrightarrow \mathbb{R}$ be continuous and define

$$d(x, y) = \max_{t \in [a,b]} |x(t) - y(t)|.$$

Show that d is a metric on X.

(3) The diameter $\delta(A)$ of a nonempty set A in a metric space (X, d) is defined to be

$$\delta(A) = \sup_{x,y \in A} d(x, y).$$

A is said to be bounded if $\delta(A) < \infty$. Show that $A \subset B$ implies $\delta(A) \leq \delta(B)$.

(4) Show that $\delta(A) = 0$ if and only if A consists of a single point.