

Discrete Mathematics
Problem Set 6
Graphs and Functions

1. Let $f : A \rightarrow B$ be an invertible function. Then show that

(a) $f^{-1} \circ f = I_A$

(b) $f \circ f^{-1} = I_B$

where $I_A : A \rightarrow A$ is the identity map defined by $I_A(x) = x$ for each $x \in A$.

2. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be bijective. Show that

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

3. Suppose that $f : A \rightarrow B$ and $g : B \rightarrow C$ are functions. Prove that if $g \circ f$ is injective, then f is injective; prove that if $g \circ f$ is surjective, then g is surjective. Hence we can conclude that if $g \circ f$ is bijective, then f is injective and g is surjective. Give an example to show that the converse of this last statement does not hold.