

Discrete Mathematics
Problem Set 5
Sets

1. What are these sets? Write them explicitly, listing their members.

(a) $\{\{2, 4, 6\}, \{6, 4\}, \} \cap \{4, 6, 8\}$

(b) $\wp(\{7, 8, 9\}) - \wp(\{7, 9\})$

(c) $\wp(\emptyset)$

(d) $\{1, 3, 5\} \times \{0\}$

(e) $\{2, 4, 6\} \times \emptyset$

(f) $\wp(\{0\}) \times \wp(\{1\})$

(g) $\wp(\wp(\{2\}))$

2. Prove the distributive law

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

3. Show that if A is a finite set with $|A| = n$, then $|\wp(A)| = 2^n$.

Hint: Consider the Taylor series expansion of $f(x) = x^n$ at 1.

4. If $|A| = n$, what is $|\wp(A) - \{\{x\} : x \in A\}|$?

5. Prove that if A, B, C, D are sets such that $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$.

6. Prove the following De Morgan's law:

(a) $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap B)^c = A^c \cup B^c$

7. Prove the following.

(a) $A \cap (A \cup B) = A$

(b) $A - (B \cap C) = (A - B) \cup (A - C)$