

Discrete Mathematics
Problem Set 3
Proof by Mathematical Induction

1. Use the formula $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ to find a closed form expression for

$$\sum_{i=0}^{n-1} (2i + 1)$$

2. (a) Using \sum notation, write an expression for the sum of the first n odd powers of 2 (that is, the sum of $2^1, 2^3$, and so on). Prove by induction that the value of this sum is $\frac{2}{3}(4^n - 1)$.
- (b) Using \prod notation, write an expression for the product of the first n negative powers of 2 (that is, the product of $2^{-1}, 2^{-2}$, and so on). What is its value?
3. Prove by induction that for any $n \geq 1$,

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

4. Prove by induction that for any $n \geq 1$,

$$\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$$

5. Prove by induction that for any $n \geq 1$,

(a) $\sum_{i=1}^n 2^{i-1} \cdot i = 2^n(n-1) + 1$.

(b) $\sum_{i=1}^n 2^{i-1} \cdot i^2 = 2^n(n^2 - 2n + 3) - 3$.