Discrete Mathematics
Problem Set 10
Quantificational Logic

1. Write as quantificational formulae. Use $L(x, y)$ to denote that x loves y.:
   (a) There is somebody who loves everybody.
   (b) There is somebody who loves no one.
   (c) There is no person who loves everyone.

2. Write the following statements using quantificational logic. Use $S(x)$ to denote that x is a student and $H(x)$ to denote that x is happy. The universe is the set of all people.
   (a) Every student is happy.
   (b) Not every student is happy.
   (c) No student is happy.
   (d) There are exactly two unhappy people, at least one of whom is a student.

3. Consider the formula

$$\exists x \exists y \exists z (P(x, y) \wedge P(z, y) \wedge P(x, z) \wedge \neg P(z, x))$$

   Under each of these interpretations, is this formula true? In each case, R is the relation corresponding to P.
   (a) $U = \mathbb{N}$, $R = \{(x, y) : x < y\}$
   (b) $U = \mathbb{N}$, $R = \{(x, x + 1) : x \geq 0\}$.
   (c) U=the set of all bit strings, $R = \{(x, y) : x \text{ is lexicographically earlier than } y\}$

(d) U=the set of all bit strings, $R = \{(x, y) : y = x0 \text{ or } y = x1\}$

(e) $U = \wp(\mathbb{N})$, $R = \{(A, B) : A \subseteq B\}$

4. Using the binary predicate $\in$ for set membership and the binary predicate $\subseteq$ for the subset relation, write formulae stating these basic properties of set membership.

   (a) Any two sets have a union.

   (b) Every set has a compliment.

   (c) Any member of a subset of a set is a member of that set.

   (d) There is a set which has no members and is a subset of every set.

   (e) The powerset of any set exists.