Discrete Mathematics
Problem Set 1
The Pigeonhole Principle

1. Show that in any group of people, two of them have the same number of friends in the group. The assumptions here are that no one is a friend of him- or herself, everyone has at least one frined in the group, and friendship is symmetrical i.e. if x is a friend of y then y is a friend of x.

2. Show that in any group of 25 people, some three of them must have birthdays in the same month.

3. A collection of coins contains six different denominations: pennies, nickels, dimes, quarters, half-dollars, and dollars. How many coins must the collection contain to guarantee that at least 100 of the coins are of the same denomination?

4. Twenty-five people go to daily yoga classes at the same gym, which offers eight classes every day. Each attendee wears either a blue, red, or green shirt to class. Show that on a given day, there is at least on class in which two people are wearing the same color shirt.

5. Show that if four distinct integers are chosen between 1 and 60 inclusive (meaning integers that are greater than or equal to 1 and less than or equal to 60), some two of them must differ by at most 19. (Hint: Try to prove it by contradiction i.e. first assume that any two them differ by more than 19 and then see if this assumption leads to a contradiction.)

6. Show that in any set of 9 positive integers, some two of them share all of the prime factors that are less than or equal to 5.