Actuarial Science
Probability
Problem Set 4
Counting and Combinatorics: Permutations and Combinations

1. Find m and n so that ${}_mP_n = \frac{9!}{6!}$.

   Solution. $m = 9$ and $n = 3$.

2. How many four-letter code words can be formed using a standard 26-letter alphabet

   (a) if repetition is allowed?

   (b) if repetition is not allowed?

   Solution: (a) $26 \times 26 \times 26 \times 26 = 456976$.

   (b) ${}_{26}P_4 = 26 \times 25 \times 24 \times 23 = 358800$.

3. Certain automobile license plates consist of a sequence of three letters followed by three digits.

   (a) If no repetitions of letters are permitted, how many license plates are there?

   (b) If no letters and no digits are repeated, how many license plates are possible?

   Solution. (a) ${}_{26}P_3 \times 10 \times 10 \times 10 = 15600000$.

   (b) ${}_{26}P_3 \times {}_{10}P_3 = 11232000$.

4. A combination lock has 40 numbers on it.

(a) How many different three-number combinations can be made?

(b) How many different combinations are there if the three numbers are different?

Solution. (a) $40 \times 40 \times 40 = 64000$.

(b) ${}_{40}P_3 = 40 \times 39 \times 38 = 59280$.

5. (a) Miss Murphy wants to seat 12 of her students in a row for a class picture. How many different seating arrangements are there?

   (b) 7 of Miss Murphy's students are girls and 5 are boys. In how many different ways can she seat the 7 girls together on the left, and then the 5 boys together on the right?

   Solution. (a) $12! = 479001600$.

   (b) $7! \times 5! = 604800$.

6. Using the digits 1, 3, 5, 7, and 9, with no repetitions of the digits, how many

   (a) one-digit numbers can be made?

   (b) two-digit numbers can be made?

   (c) three-digit numbers can be made?

   (d) four-digit numbers can be made?

   Solution. (a) ${}_5P_1 = \frac{5!}{4!} = 5$.

   (b) ${}_5P_2 = \frac{5!}{3!} = 20$.

   (c) ${}_5P_3 = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$.

   (d) ${}_5P_4 = 5! = 120$.

7. There are five members of the Math Club. In how many ways can the positions of officers, a president and a treasurer, be chosen?

   Solution. There are 5 ways to choose a president. For each choice of a president, there are 4 ways to choose a treasurer. Hence, the answer to the question is $5 \times 4 = 20$ ways.

8. (a) A baseball team has nine players. Find the number of ways the manager can arrange the batting order.

(b) Find the number of ways of choosing three initials from the alphabet if none of the letters can be repeated. Name initials such as MBF and BMF are considered different.

Solution. (a) $9! = 362880$.

(b) ${}_{26}P_3 = 26 \times 25 \times 24 = 15600$.

9. Find m and n so that ${}_mC_n = 13$.

   Solution. $m = 13$ and $n = 1$ or $m = 13$ and $n = 12$.

10. The Library of Science Book Club offers three books from a list of 42. If you circle three choices from a list of 42 numbers on a postcard, how many possible choices are there?

    Solution. The order of choices of three books does not matter because, for example, circling 1 4 18 or 18 1 4 in that order results in exactly the same set of three books, Hence there are ${}_{42}C_3 = 11480$ possible choices.

11. At the beginning of the second quarter of a mathematics class for elementary school teachers, each of the class's 25 students shook hands with each of the other students exactly once. How many handshakes took place?

    Solution. It is the same as the number of ways to choose two students out of 25 with no regard to an order, i.e ${}_{25}C_2 = \frac{25\times24}{2} = 300$.

12. There are five members of the math club. In how many ways can the two-person Social Committee be chosen?

    Solution. ${}_5C_2 = 10$.

13. A consumer group plans to select 2 televisions from a shipment of 8 to check the picture quality. In how many ways can they choose 2 televisions?

    Solution. ${}_8C_2 = 28$.

14. A school has 30 teachers. In how many ways can the school choose 3 people to attend a national meeting?

    Solution. ${}_{30}C_3 = 4060$.

15. Which is usually greater the number of combinations of a set of objects or the number of permutations?

    Solution. Since $n! \geq 1$, ${}_mP_n = n!{}_mC_n \geq {}_mC_n$.

16. How may different 12-person juries can be chosen from a pool of 20 jurors?

    Solution. ${}_{20}C_{12} = 125970$.

17. A jeweler has 15 different sized pearls to string on a circular band. In how many ways can this be done?

    Solution. $\frac{(15-1)!}{2} = \frac{14!}{2} = 43589145600$.

18. Four teachers and four students are seated in a circular discussion group. Find the number of ways this can be done if teachers and students must be seated alternately.

    Solution. The number of ways to seat four teachers at a round table is $(4-1)! = 3!$. Suppose that there is a seat for a student between two teachers, totaling four seats for students at the round table. For each choice of teachers' seatings, there are 4! ways of seating students between teachers. Therefore, There are $3! \times 4! = 6 \times 24 = 144$ ways the required seatings can be done.