Actuarial Science
Probability
Problem Set 3
Counting and Combinatorics:
The Fundamental Principle of Counting

1. If each of the 10 digits in chosen at random, how many ways can you choose the following numbers?

   (a) A two-digit code number, repeated digits permitted.
   Solution. Since the two-digit code number would permit 0 for the first digit, $10 \times 10 = 100$ ways.

   (b) A three-digit identification card number, for which the first digit cannot be a 0.
   Solution. $9 \times 10 \times 10 = 900$.

   (c) A four-digit bicycle lock number, which no digit can be used twice.
   Solution. $10 \times 9 \times 8 \times 7 = 5,040$.

   (d) A five-digit zip code number, with the first-digit not zero.
   Solution. $9 \times 10 \times 10 \times 10 \times 10 = 90,000$.

2. (a) If eight horses are entered in a race and three finishing places are considered, how many finishing orders can they finish? Assume no ties.
   Solution. $8 \times 6 = 336$.

   (b) If the top three horses are Lucky One, Lucky Two, and Lucky Three, in how many possible orders can they finish?
   Solution. $3 \times 2 \times 1 = 6$.

3. You are taking 3 shirts (red, blue, yellow) and 2 pairs of pants (tan, gray) on a trip. How many different choices of outfits do you have?

   Solution. $3 \times 2 = 6$.

4. A club has 10 members. In how many ways can the club choose a president and a vice-resident if everyone is eligible?

   Solution. $10 \times 9 = 90$.

5. In a medical study, patients are classified according to whether they have blood type A, B, AB, or O, and also according to whether their blood pressure is low (L), normal (N), or high (H). Use a tree diagram to represent the various outcomes that can occur.

   Solution. For each of the 4 blood types, there can be three levels of blood pressure. So, the number of all possible outcomes is $4 \times 3 = 12$.

6. If a travel agency offers special weekend trips to 12 different cities, by air, rail, or bus, in how many different ways can such a trip be arranged?

   Solution. $12 \times 3 = 36$.

7. If twenty paintings are entered in an art show, in how many ways can the judges award a first prize and a second prize?

   Solution. $20 \times 19 = 380$.

8. In how many ways can the 52 members of a labor union choose a president, a vice-president, a secretary, and a treasurer?

   Solution. $52 \times 51 \times 50 \times 49 = 6,497,400$.

9. Find the number of ways in which four of ten new movies can be ranked first, second, third, and fourth according to their attendance figures for the first six months.

   Solution. $10 \times 9 \times 8 \times 7 = 5,040$.

10. How many ways are there to seat 10 people, consisting of 5 couples, in a row of seats (10 seats wide) if all couples are to get adjacent seats?

    Solution. There are $5! = 120$ ways of arranging five couples in a row. For each couple, there are $2! = 2$ ways to seat them. Therefore,

the total number of ways to seat the 10 people in a row as required is

$$2! \times 2! \times 2! \times 2! \times 2! \times 5! = 2^5 \times 120 = 3,840$$