Differential Equations
Problem Set 8
Non-Homogeneous Second-Order Differential Equations:
The Method of Undetermined Coefficients

1. Determine the general solution of the given differential equation.

   (a) $\ddot{x} - 2\dot{x} - 3x = -3te^{-t}$

   (b) $\ddot{x} + 2\dot{x} = 3 + 4\sin(2t)$

   (c) $\ddot{x} + x = 3\sin(2t) + t\cos(2t)$

   (d) $\ddot{x} + \dot{x} - 6x = 12e^{3t} + 12e^{-2t}$

   (e) $\ddot{x} + 3\dot{x} = 2t^4 + t^2e^{-3t} + \sin(3t)$

   Hint: Suppose that $X_1(t)$, $X_2(t)$, $X_3(t)$ are solutions of

   $$\begin{aligned}\ddot{x} + 3\dot{x} &= 2t^4\\ \ddot{x} + 3\dot{x} &= t^2e^{-3t}\\ \ddot{x} + 3\dot{x} &= \sin(3t)\end{aligned}$$

   respectively. Then $X_1(t) + X_2(t) + X_3(t)$ is a solution of the equation

   $$\ddot{x} + 3\dot{x} = 2t^4 + t^2e^{-3t} + \sin(3t)$$

   (f) $\dddot{x} - 3\ddot{x} + 3\dot{x} - 1 = 4e^t$

   Hint: You probably haven't dealt with a linear differential equation of order higher than 2. However the principle of solving the general solution of a Homogeneous higher order linear differential equation is essentially the same as the second-order case, namely using the characteristic equation.

2. Find the solution of the given initial value problem.

   (a) $\ddddot{x} + 4x = t^2 + 3e^t$ with $x(0) = 0$, $\dot{x}(0) = 2$

   (b) $\ddot{x} + 4x = 3\sin(2t)$ with $x(0) = 2$, $\dot{x}(0) = -1$

   (c) $\ddot{x} + 2\dot{x} + 5x = 4e^{-t}\cos(2t)$ with $x(0) = 1$, $\dot{x}(0) = 0$