Differential Equations
Problem Set 6
Harmonic Motion

1. Consider harmonic oscillator

$$m\ddot{x} + c\dot{x} + kx = 0$$

with mass m, damping coefficient c, and spring constant (also called stiffness) k. For the values specified along with given initial conditions, solve the equation of harmonic oscillator.

   (a) $m = 1$, $c = 8$, $k = 7$ with initial conditions $x(0) = -1$, $\dot{x}(0) = 5$
   (b) $m = 1$, $c = 6$, $k = 8$ with initial conditions $x(0) = 1$, $\dot{x}(0) = 0$
   (c) $m = 1$, $c = 4$, $k = 5$ with initial conditions $x(0) = 1$, $\dot{x}(0) = 0$
   (d) $m = 1$, $c = 0$, $k = 8$ with initial conditions $x(0) = 1$, $\dot{x}(0) = 4$
   (e) $m = 2$, $c = 3$, $k = 1$ with initial conditions $x(0) = 0$, $\dot{x}(0) = 3$
   (f) $m = 9$, $c = 6$, $k = 1$ with initial conditions $x(0) = 1$, $\dot{x}(0) = 1$
   (g) $m = 2$, $c = 0$, $k = 3$ with initial conditions $x(0) = 2$, $\dot{x}(0) = -3$
   (h) $m = 2$, $c = 1$, $k = 3$ with initial conditions $x(0) = 0$, $\dot{x}(0) = -3$

2. Harmonic oscillator

$$m\ddot{x} + c\dot{x} + kx = 0$$

has one real solution $e^{-\frac{c}{2m}t}$ if $c^2 - 4mk = 0$. Show that $te^{-\frac{c}{2m}t}$ is also a solution.