Differential Equations
Problem Set 2
Homogeneous Differential Equations

1. Solve each of the following Homogeneous differential equations.
   (a) $\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$
   (b) $\frac{dy}{dx} = \frac{y-4x}{x-y}$
   (c) $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$
   (d) $xy\frac{dy}{dx} = x^2 + y^2$
   (e) $\frac{dy}{dx} = \frac{y-x}{x+y}$

2. Consider a reflector which is a surfaces of revolution. The reflector reflects light rays that come from a point source O on the axis of rotation. We would like to know what kind of surface the reflector should be in order for the reflected light rays to be parallel to the axis of rotation. In Figure 1, the curve in blue is the cross section of the reflector on the xy-plane and the x-axis is the axis of rotation. $\overline{AT}$ is line tangent to the cross section at $M(x, y)$ and $\overline{MN}$ is line normal to the cross section at M. If we let $\angle OAM = \alpha$, then $\angle SMT = \alpha$. By the law of reflection, the angle of incidence $\angle OMN$ and the angle of reflection $\angle NMS$ are the same, consequently $\angle OMA = \alpha$. Since $\triangle\, OAM$ is an isosceles triangle, $AO = OM$. $AO = AP - OP = \frac{y}{y'} - x$ and $OM = \sqrt{x^2 + y^2}$. Therefore we have the differential equation

$$\frac{y}{y'} - x = \sqrt{x^2 + y^2} \tag{1}$$

   This is a homogeneous differential equation.

(a) solve the equation (1).The solution is the parabola $y^2 = 2C\left(x + \frac{C}{2}\right)$ where C is a constant. Hence the desired surface of revolution (the shape of mirror) is the paraboloid $y^2 + z^2 = 2C\left(x + \frac{C}{2}\right)$.

(b) If the diameter of the parabolic mirror is d and the depth is h, determine the constant C and write the resulting equation of parabola.

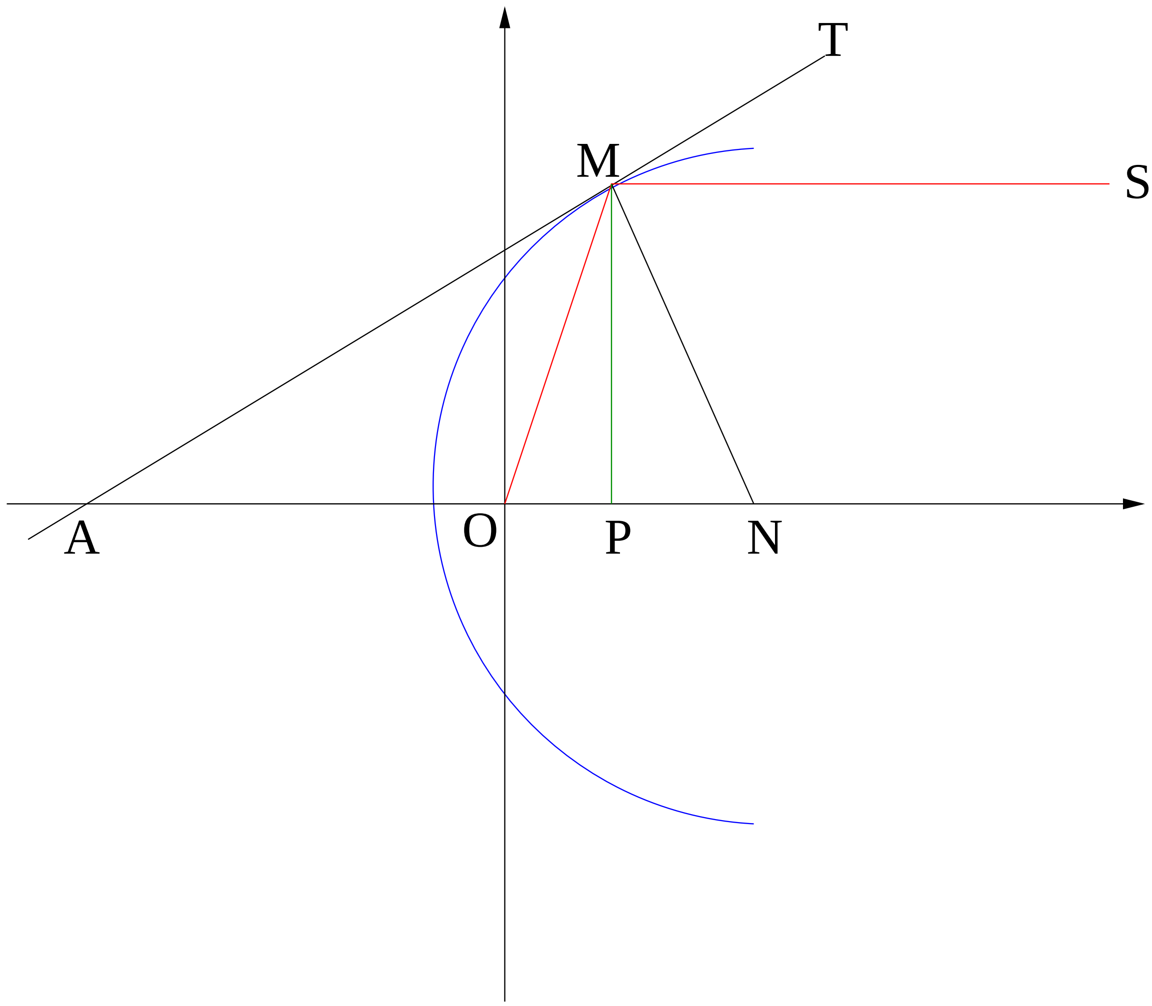

Figure 1: Reflector