Differential Equations
Problem Set 14
The Laplace Transform: Convolution

1. With the aid of the inverse transform

$$\mathscr{L}^{-1}\left\{\frac{1}{(s^2+k^2)^2}\right\} = \frac{1}{2k^3}(\sin kt - kt\cos kt)$$

   show that

$$\mathscr{L}^{-1}\left\{\frac{8}{(s^2+1)^3}\right\} = 3(\sin t - t\cos t) - t^2\sin t$$

2. Solve the following differential equations.

   (a) $\ddot{X}(t) - k^2X(t) = F(t)$ if $X(0) = \dot{X}(0) = 0$ $(k \neq 0)$

   (b) $\ddot{X}(t) - 2k\dot{X}(t) + k^2X(t) = F(t)$

   (c) $2\ddot{X}(t) - 3\dot{X}(t) - 2X(t) = F(t)$ if $X(0) = \dot{X}(0) = 0$

   (d) $\dddot{X}(t) - \dot{X}(t) = F(t)$

3. Solve for $X(t)$:

$$\int_0^t X(\tau)d\tau - \dot{X}(t) = t,\ X(0) = 2$$

4. Solve the integral equations.

   (a) $X(t) = a\sin t - 2\int_0^t X(\tau)\cos(t-\tau)d\tau$

   (b) $X(t) = a\sin t + \int_0^t X(\tau)\sin(t-\tau)d\tau$