Differential Equations
Problem Set 11
The Laplace Transform:
Transforms of Derivatives

1. Obtain these transforms with the aid of

$$\mathscr{L}\{F^{(n+1)}(t)\} = s^{n+1}\mathscr{L}\{F(t)\} - s^n F(0) - s^{n-1}F'(0) - \cdots - F^{(n)}(0)$$

   (a) $\mathscr{L}\{\cos kt\} = \frac{s}{s^2+k^2}$

   (b) $\mathscr{L}\{\sinh kt\} = \frac{k}{s^2-k^2}$

   (c) $\mathscr{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$

   (d) $\mathscr{L}\{te^{kt}\} = \frac{1}{(s-k)^2}$

2. The Gamma function is defined by

$$\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}\,dx \ (r > 0)$$

   (a) Use integration by parts to show that the Gamma function satisfies

$$\Gamma(r+1) = r\Gamma(r)$$

   (b) Show that $\Gamma(1) = 1$, and hence that $\Gamma(n+1) = n!$ when $n = 1, 2, 3, \cdots$.

   (c) Given that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$, find $\Gamma\left(\frac{3}{2}\right)$ and $\Gamma\left(\frac{5}{2}\right)$.