

FUNCTIONS OF A COMPLEX VARIABLE
PROBLEM SET 6: APPLICATIONS OF RESIDUES

(1) Use residues to evaluate the improper integrals.

(a) $\int_0^{\infty} \frac{dx}{x^2+1}$.

(b) $\int_0^{\infty} \frac{dx}{x^4+1}$.

(c) $\int_0^{\infty} \frac{x^2 dx}{x^6+1}$.

(d) $\int_0^{\infty} \frac{\cos ax}{x^2+1} dx$ ($a > 0$).

(e) $\int_{-\infty}^{\infty} \frac{\cos x dx}{(x^2+a^2)(x^2+b^2)}$ ($a > b > 0$).

(2) Use residues to evaluate the definite integral

$$\int_{-\pi}^{\pi} \frac{d\theta}{1 + \sin^2 \theta}.$$

(3) Use residues to evaluate the definite integral

$$\int_0^{2\pi} \frac{d\theta}{1 + a \cos \theta} \quad (-1 < a < 1).$$