FUNCTIONS OF A COMPLEX VARIABLE
PROBLEM SET 5: RESIDUES, CAUCHY'S RESIDUE THEOREM

(1) In each case write the principal part of the function at its isolated singularity. Determine if that point is a pole, an essential singularity, or a removable singularity of the given function.
   (a) $ze^{\frac{1}{z}}$;
   (b) $\frac{z^2}{1+z}$;
   (c) $\frac{\sin z}{z}$;
   (d) $\frac{\cos z}{z}$.

(2) Show that all the singularities of each of the following functions are poles. Determine the order of m of each pole and the corresponding residue B.
   (a) $\frac{z+1}{z^2-2z}$;
   (b) $\frac{1-e^{2z}}{z^4}$;
   (c) $\frac{e^{2z}}{(z-1)^2}$;
   (d) $\frac{e^z}{z^2+\pi^2}$.

(3) Evaluate the contour integral

$$\int_C \frac{3z^3+2}{(z-1)(z^2+9)}dz$$

taken conterclockwise around the circle
   (a) $|z-2|=2$;
   (b) $|z|=4$.

(4) Evaluate the contour integral

$$\int_C \frac{dz}{z^3(z+4)}$$

taken counterclockwise aroundthe circle
   (a) $|z|=2$;
   (b) $|z+2|=3$.

(5) Let C be the circle $|z| = 2$ described in the positive sense and evaluate the contour integral
  (a) $\int_C \tan z dz$;
  (b) $\int_C \frac{dz}{\sinh 2z}$;
  (c) $\int_C \frac{\cosh \pi z dz}{z(z^2+1)}$.
(6) Use Cauchy's residue theorem to evaluate the integral of each function around the circle $|z| = 3$ in the positive sense:
  (a) $\frac{\exp(-z)}{z^2}$
  (b) $\frac{\exp(-z)}{(z-1)^2}$
  (c) $z^2 \exp\left(\frac{1}{z}\right)$
  (d) $\frac{z+1}{z^2-2z}$
(7) Find the value of the integral
$$\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz,$$
taken counterclockwise around the circle
  (a) $|z-2| = 2$
  (b) $|z| = 4$
(8) Find the value of the integral
$$\int_C \frac{dz}{z^3(z+4)},$$
taken counterclockwise around the circle
  (a) $|z| = 2$
  (b) $|z+2| = 3$
(9) Evaluate the integral
$$\int_C \frac{\cosh \pi z}{z(z^2+1)} dz$$
where C is the circle $|z| = 2$, described in the postive sense.